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RESEARCH PROJECT INITIATION

Date: October 3, 1975

Project Title: Integral-Representation Methods for Solution to Viscous Flow Problems

Project No: E-16-673

Principal Investigator Dr. James C. Wu

Sponsor: National Science Foundation

Agreement Period: From 9/15/75 Until 2/28/78
24 months budget period plus 6 months for submission of required reports, etc.

Type Agreement: Grant No. ENG74-24719

Amount: $55,900 NSF
4,226 GIT E-16-363
$60,126 Total

Reports Required:

Annual Letter Technical, Final Report

Sponsor Contact Person (s):

Administrative Matters
thru OCA

Mr. Gaylord L. Ellis
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SPONSORED PROJECT TERMINATION

Date: 10/17/78

Project Title: Integral-Representation Methods for Solution to Viscous Flow Problems.

Project No: E-16-673

Project Director: Dr. J. C. Wu

Sponsor: National Science Foundation

Effective Termination Date: 2/28/78

Clearance of Accounting Charges: 2/28/78

Grant/Contract Closeout Actions Remaining:

- _ Final Invoice and Closing Documents
- x Final Fiscal Report
- _ Final Report of Inventions
- _ Govt. Property Inventory & Related Certificate
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E-16-673

NSF GRANT No. ENG 74-24719

Annual Progress Report

for the Period

September 15, 1975 to September 14, 1976

INTEGRAL-REPRESENTATION METHODS FOR SOLUTION TO VISCOUS FLOW PROBLEMS

Prepared by

James C. Wu, Professor

School of Aeorspace Engineering

Georgia Institute of Technology

for

The Director, Fluid Mechanics Program

Engineering Mechanics Section

Engineering Division

National Science Foundation

Washington, D. C. 20550

SUMMARY

During the first year of this research program, a systematic study has yielded integral representations for both kinetic and kinematic aspects of flow problems under very general circumstances--for transient and steady, compressible and incompressible, viscous and potential, turbulent and laminar flows. Implementation of numerical procedures for several types of flow problems has been explored. For the steady, incompressible, viscous flow problem, a procedure has been developed, calibrated, refined, and applied to several test problems.

The integral representations are exact equivalences of the more familiar differential equations of motion. Numerical procedures based on the integral representations, however, have been shown to possess distinguishing features not available to those based on differential formulations or the usual finite-element formulations. These features have been shown to lead to several important attributes, in particular, the ability to confine the solution field in incompressible flows to regions of viscous flow and the ability to have the confined solution field segmented and each segment treated individually. It has been demonstrated by analyses and by numerical illustrations, in the case of steady incompressible flows, that these attributes enable the integral-representation method to transcend the limitations of prevailing methods and to be particularly well suited for flows at high Reynolds numbers.

RESEARCH ACTIVITIES AND RESULTS

During the first year of this research effort, a systematic study of integral representations for flows under very general circumstances has been made, and the implementation of an integral-representation method for the solution of steady incompressible viscous flows has been carried out. Predicted advantages of the method have been demonstrated by the solution of several sample problems of practical interest. Considerable experience has been gained and additional problems of great complexity are being solved using this new method. Specific tasks that have been carried out are described below.

(1) Further extension of the integral-representation method.

With the integral-representation method, the kinematic and kinetic aspects of the flow problem are formulated as integral representations for field variables. There are several alternative methods of establishing integral representations in somewhat different forms, each possessing unique features of interest. A systematic study of formulation of integral representations for elliptic and parabolic differential equations has been completed. Application of the integral-representation method to various types of flow problems--potential and viscous, time-dependent and steady, incompressible and compressible--has been considered. The results of this study are described in a recent article by the principal investigator. (No. 1 in the attached list of publications.) The availability of integral representations under very general circumstances is expected to be a significant factor in future solutions of flow problems, including but not limited to steady incompressible flows which form the focal problem for numerical studies in the present research.

A difficult aspect of previous numerical methods for the solution of viscous flow problems has been the establishment of "extraneous boundary conditions". The present research showed that the use of integral representation eliminates this difficulty. A comprehensive discussion of this point concerning the kinematic extraneous (vorticity) boundary condition is presented in Article 2 of the attached list of publications. The method described in that article has been extended and successfully used in the kinetic part (extraneous vorticity gradient boundary condition) of the numerical procedure.

(2) Development of analytical expressions for geometric functions.

The solution procedure based on integral representations requires the evaluation of geometric functions which are dependent on (a) the relative position of the point at which a field variable is to be computed and the position of the element contributing to the field variable, (b) the shape of the element, and (c) the order of the interpolation function used for the element. An exact analytical expression has been derived during the reporting year for these geometric functions. This expression is valid for elements of any polygonal shape with any number of sides and with any order of interpolation function. The availability of this general analytic expression makes it possible to represent complex boundary geometries in a convenient manner, to compute field variables in a highly efficient and accurate manner, and to establish optimum solution procedures as described below.

(3) Establishment of optimum solution procedures.

The new integral representation formulation of the viscous flow problem is widely different in form from the differential equations or the variational principle or Galerkin formulations that form the basis of finite difference and finite element methods. This difference has led to many drastic advantages, for example, the confinement of the solution to the viscous region, and at the same time has necessitated the development of entirely new numerical procedures. Exhaustive analyses and numerical experimentation have been carried out to establish optimum solution procedures. The major conclusions reached are: (a) Second order triangular elements are among the most suitable ones to use in conjunction with the integral-representation method. They lead to highly accurate solutions and require relatively small amounts of computing effort for a wide range of flow conditions. (b) An iterative procedure, in which velocity and vorticity distributions are alternately computed using values obtained in the previous iteration, has been shown to be highly efficient. (c) High Reynolds number solutions may be obtained efficiently by using low Reynolds number solutions for the same geometry as an initial solution to begin the iteration solution.

Further refinements of the solution procedure are being studied. The procedure in its present stage of development, however, has been shown to be already drastically more efficient, in terms of computing effort and accuracy, than previously available methods.

(4) Numerical simulation of interior flow problems.

As a test problem for the numerical procedure that has been developed, Couette flows between parallel plates with zero, favorable, and adverse pressure gradients were treated. This problem is one-dimensional and has a well known exact solution. In the present study, however, the problem was treated as a two-dimensional flow inside a rectangular region. The velocity at the boundary of the rectangle, known from the exact solution, is the only prescribed condition in the numerical procedure. Considerable computer experimentation was carried out on the basis of this test problem.

Solutions were also obtained for a two-dimensional square cavity with a moving flat plate above it. The results for Reynolds numbers 1, 100, and 600 were presented in Article 3 of the list of publications and compared with available solutions obtained using various other methods. The present method was shown to possess remarkable solution speed and accuracy, and to simulate physical processes more accurately than some other methods.

In addition to the above itemized activities, numerical results have also been obtained for two-dimensional stagnation point flow. A study has been made to implement techniques of solution field segmentation. Work completed during the first year of this research has been restricted to the kinematic aspect of the problem. The conclusion reached was that segmentation techniques offer very drastic reductions in computing effort with no adverse effects on solution accuracy. Studies have also been made to extend the integral-representation method to turbulent flows. Very encouraging results were obtained using the integral representation method in conjunction with a two-equation turbulence model.

RESEARCH ACCOMPLISHMENT

Many separated viscous problems of practical concern remain today beyond the scope of prevailing finite-difference and finite-element methods. This limitation is primarily due to the often prohibitive amounts of computing effort required to obtain reasonably accurate solutions and is particularly acute for flows at moderate and high Reynolds numbers. In the present research, the basic cause of the obstacle of excessive computing time has been examined, analyzed, and alleviated for incompressible flows. As a result, a new method which transcends the limitation of prevailing methods and which is well suited for separated viscous flows at high Reynolds numbers has been established.

For a large class of high Reynolds number flow problems—generally those requiring excessive computing effort--there exists in the flowfield a small region in which effects of viscous forces are important and gradients of field variables are large. This small viscous region is embedded in a much larger region of essentially potential flow in which the gradients are much smaller in comparison. In other words, the length scale for the viscous region is much smaller than that for the potential region. Prevailing finite-difference and finite-elemnet methods are based on flow equations formulated as differential relations and relations obtainable through the concepts of variational principles, weighted residuals, or Galerkin's approach. In the kinematic aspect of the incompressible flow problem, the differential equations (and their finite-element derivatives) are elliptic. As a consequence, a solution involving the entire flowfield, inclusive of the viscous and the potential regions, is required in the prevailing methods. This means that the two regions involving vastly different length scales must be treated simultaneously. It then becomes extremely difficult to devise a data grid which provides sufficient solution resolution in the viscous region and yet does not contain an excessive number of data points--and hence requires excessive computing time and data storage--in the potential flow region.

A systematic study of alternative formulations of flow equations has led to integral representations of field variables that are exact equivalences of the differential equations of motion but that allow the evaluation of field variables using explicit numerical procedures. Both the kinetic aspect and the kinematic aspect of flow problems have been expressed as integral representations under rather general circumstances--for transient and steady, compressible and incompressible, viscous and potential, turbulent and laminar flows. The distinguishing feature of integral representations, that it permits explicit evaluation of field variables, permits the solution field for incompressible flows to be confined to the viscous region. The obstacle of excessive computing needs, caused by simultaneous treatment of viscous and potential regions as discussed earlier, is therefore alleviated.

For the problem of steady incompressible separated laminar flow, which forms the focal problem for numerical studies in the present research problem, a numerical method based on integral representations for the velocity and vorticity vectors has been developed, calibrated, and used in studies of several test problems of practical interest. Predicted advantages of the new method have been verified by analysis and by numerical experimentation. In particular, it has been demonstrated that the method transcends the limitations of prevailing methods and is particularly well suited for flows involving appreciable flow separations at high Reynolds numbers.

The successful implementation of the integral representation method for the steady, incompressible, laminar flow problem is highly significant since it demonstrated not only the validity but also the striking advantages offered by this entirely new approach to a very complex problem. These advantages are not limited to steady, incompressible, laminar flows. Indeed, a critical examination of the integral representations for incompressible time-dependent flows and turbulent flow reveals no fundamental difficulties in the implementation of this new method for these types of flows.* It is concluded that a much brighter prospect of accurate quantitative prediction of fairly complex viscous flow problems at high Reynolds numbers is now in sight.

*The study of turbulent flows of course requires a suitable description of the turbulence phenomena. A limited amount of research has been conducted using the integral representation method in conjunction with a two-equation turbulence model. Very encouraging results have been obtained.

PERSONNEL SUPPORTED

1. Professor James C. Wu

   Dr. Wu directed this project as principal investigator and performed research described in Item 1 of the section on Research Activities and Results of this report. He also contributed substantially to research described in Items 2 and 4 of that section.

2. Dr. M. M. Wahbah

   Dr. Wahbah contributed to this program initially as a graduate student and later as a post-doctoral fellow. He is the major contributor to the research described in Items 2, 3, and 4 of the Section on Research Activities and Results of this report.

3. K. Balasubramanian

   Mr. Balasubramanian assisted the principal investigator and Dr. Wahbah in the numerical studies performed. His Ph.D. thesis research is directly related to this research program.

4. Y. M. A. Rizk

   Mr. Rizk is studying the integral-representation method for time-dependent flows. He has obtained numerical solutions for two-dimensional stagnation point flows using the integral-representation method.

PUBLICATIONS

1. J. C. Wu, "Finite Element Solution of Flow Problems Using Integral Representations," Proceedings of the Second International Symposium on Finite Element Methods in Flow Problems, pp. 205-216, International Centre for Computer Aided Design, June, 1976.

2. J. C. Wu and M. M. Wahbah, "Numerical Solution of Viscous Flow Equations Using Integral Representations," Proceedings of the Fifth International Conference on Numerical Methods in Fluid Dynamics, Springer-Verlag, in print.

3. J. C. Wu, "Numerical Boundary Conditions for Viscous Flow Problems," AIAA Journal, pp. 1042-1049, August, 1976.

4. J. C. Wu, "The Prospects for Computational Fluid Dynamics," Proceedings of 1976 Viscous Flow Symposium, Lockheed-Georgia Company, 1976 - In preparation.

Copies of the first three articles are attached.

RELATED EVENTS

The principal investigator has been invited to actively cooperate on a NATO project in Turkey that is related to this research. He has also been invited to present lectures at three different specialist seminars and workshops on topics originating in part from this research program. A local aircraft industry—the Lockheed-Georgia Company has requested some of the research results and is presently utilizing parts of subroutines prepared in the course of this research. Several governmental and educational institutions, both in the U.S. and abroad, have initiated research work utilizing integral representations that have been developed here.

NATIONAL SCIENCE FOUNDATION
Washington, D.C. 20550

FINAL PROJECT REPORT
NSF FORM 98A

PLEASE READ INSTRUCTIONS ON REVERSE BEFORE COMPLETING

PART I–PROJECT IDENTIFICATION INFORMATION

1. Institution and Address	2. NSF Program	3. NSF Award Number
Georgia Institute of Technology Atlanta, Georgia 30332	Fluid Mechanics	ENG 74-24719
	4. Award Period From 9/15/75 To 2/28/78	5. Cumulative Award Amount $55,900

6. Project Title

Integral-Representation Methods for Solution to Viscous Flow Problems

PART II–SUMMARY OF COMPLETED PROJECT *(FOR PUBLIC USE)*

The primary objective of this project is to fully develop a new method for the numerical solution of steady incompressible viscous flow problems which transcends the limitations of previous methods for flows involving high Reynolds number, complex geometry, and massive separation. This new method was established on the basis of integral-representations for flow variables. Integral-representations were obtained, through a systematic study, for both kinetic and kinematics aspect of flow problems under very general circumstances -- for transient and steady, compressible and incompressible, viscous and potential, turbulent and laminar flows. Implementation of numerical procedures utilizing the integral representations for several types of flow problems has been completed. In particular, for steady incompressible viscous flows, a standardized, user-oriented package of computer code was developed and calibrated for internal two-dimensional problems. This package requires as input only the geometry and the velocity condition of the flow boundary. The anticipated advantages of superior solution efficiency, solution accuracy, and procedure universality have been conclusively demonstrated by numerical illustrations. Solutions have been obtained for a number of complex viscous flow problems involving flow separation. The application of integral-representation approach has been extended to compressible and turbulent flow problems.

PART III–TECHNICAL INFORMATION *(FOR PROGRAM MANAGEMENT USES)*

1. ITEM *(Check appropriate blocks)*	NONE	ATTACHED	PREVIOUSLY FURNISHED	TO BE FURNISHED SEPARATELY TO PROGRAM: Check (✓)	Approx. Date
a. Abstracts of Theses			x		
b. Publication Citations			x		
c. Data on Scientific Collaborators			x		
d. Information on Inventions			x		
e. Technical Description of Project and Results			x		
f. Other *(specify)*					

2. Principal Investigator/Project Director Name *(Typed)*	3. Principal Investigator/Project Director Signature	4. Date
James C. Wu		8/29/79

Room 638

INSTRUCTIONS FOR FINAL PROJECT REPORT
(NSF FORM 98A)

This report is due within 90 days after the expiration of the award. It should be submitted in two copies to:

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Washington, D.C. 20550

INSTRUCTIONS FOR PART I

These identifying data items should be the same as on the award documents.

INSTRUCTIONS FOR PART II

The summary (about 200 words) must be self-contained and intelligible to a scientifically literate reader. Without restating the project title, it should begin with a topic sentence stating the project's major thesis. The summary should include, if pertinent to the project being described, the following items:

- The primary objectives and scope of the project.
- The techniques or approaches used only to the degree necessary for comprehension.
- The findings and implications stated as concisely and informatively as possible.

This summary will be published in an annual NSF report. Authors should also be aware that the summary may be used to answer inquiries by nonscientists as to the nature and significance of the research. Scientific jargon and abbreviations should be avoided.

INSTRUCTIONS FOR PART III

Items in Part III may, but need not, be submitted with this Final Project Report. Place a check mark in the appropriate block next to each item to indicate the status of your submission.

a. Self-explanatory.
b. For publications (published and planned) include title, journal or other reference, date, and authors. Provide two copies of any reprints as they become available.
c. Scientific Collaborators: provide a list of co-investigators, research assistants and others associated with the project. Include title or status, e.g. associate professor, graduate student, etc.
d. Briefly describe any inventions which resulted from the project and the status of pending patent applications, if any.
e. Provide a technical summary of the activities and results. The information supplied in proposals for further support, updated as necessary, may be used to fulfill this requirement.
f. Include any additional material, either specifically required in the award instrument (e.g. special technical reports or products such as films, books, studies) or which you consider would be useful to the Foundation.

E16-673

NSF GRANT NO. ENG 74-24719

Final Report

INTEGRAL-REPRESENTATION METHODS FOR SOLUTION TO VISCOUS FLOW PROBLEMS

Prepared by

James C. Wu, Principal Investigator

Georgia Institute of Technology

for

The Director, Fluid Mechanics Program

Engineering Mechanics Section

Engineering Division

National Science Foundation

Washington, D.C. 20550

SUMMARY

A systematic study has yielded integral representations for both kinetic and kinematic aspects of flow problems under very general circumstances --for transient and steady, compressible and incompressible, viscous and potential, turbulent and laminar flows. Implementation of numerical procedures utilizing the integral representations for several types of flow problems has been completed. In particular, for steady incompressible viscous flows, a standardized, user-oriented package of computer code was developed and calibrated for internal two-dimensional problems. This package requires as input only the geometry and the velocity condition of the flow boundary. The anticipated advantages of superior solution efficiency, solution accuracy, and procedure universality have been conclusively demonstrated by numerical illustrations. Solutions have been obtained for a number of complex viscous flow problems involving flow separation. The application of integral-representation approach has been extended to compressible and turbulent flow problems.

RESEARCH ACTIVITIES AND RESULTS

In the Research Proposal submitted to NSF and dated December 15, 1974, the principal investigator stated the following specific objectives:

"The primary objective of the proposed research is to fully develop a new method for the solution of steady incompressible viscous flow problems, utilizing the integral-representations formulation of the Navier-Stokes equation, which transcends the limitations of previous methods for flows involving high Reynolds number and complex geometries. The secondary objectives are to further implement several solution techniques made possible by the integral-representations, including the segmentation of solution field and kinematical treatment of boundary conditions, that remove the difficulties encountered in other methods. In addition, the integral representations for compressible and time-dependent problems will be studied. The proposed research emphasizes a combined theoretical and numerical study."

These objectives have all been reached. The development of the integral representation method for the numerical solution of steady laminar incompressible flow problems has been completed. Furthermore developments of this method for unsteady flows, compressible flows, and turbulent flows have been initiated under separate projects. Specific tasks that have been carried out are outlined below:

(1). Development of exact analytical expressions for geometric functions.

The numerical quadrature of integrals involved in the integral-representation method requires the evaluation of certain geometric functions. The solution procedure developed in conjunction with the method employs polygonal finite elements to map out the solution field. Geometric coefficients associated with elements of any polygonal shape with any number of sides and with polynomial interpolation functions of any order have been expressed in closed forms. The availability of these closed form expressions made it possible to routinely accomodate complex boundary shapes, to carry out the necessary computations with superior efficiency and high accuracy, and to establish a standarized solution procedure for various types of flows.

(2). Refinement and optimization of flowfield segmentation method.

It has been shown that the integral representation method possesses the distinguishing ability of confining the solution field to the vortical region of the flow. For high Reynolds number flows where the major portion of the flowfield is non-vortical, the drastic reduction of the solution field offers great computational advantages. Segmentation of the solution field, already confined to the vortical region, is possible if the integral representation method is used. It has been shown in this research, analytically and numerically, that flowfield segmentation provides an inherent flexibility to the solution procedure and offers further drastic computational advantages.

(3). Parallel programming.

It has been shown that the segmentation of solution field enable the computation of field variables at nodes interior of each segment to be performed individually, independent of those of other segments. This ability is ideally suited for the solution of three-dimensional problems (which require enormous amounts of computation) on super computers possessing parallel programming features.

(4). Numerical simulation of internal flows.

A standarized, user-oriented computer code for solving two-dimensional internal laminar flows within arbitrarily prescribed boundaries has been developed. This computer code is prepared and documented under a separate project, using the basic knowledge gained in the present research. This code requires only the input of node locations and velocity boundary conditions. No additional efforts, such as special techniques for handling complex boundary shapes, are required. This code has been used to treat a large number of internal flow problems, including flows in rectangular cavities and inside constricted channels.

(5). Numerical simulation of external flows

A large number of unsteady external flow problems have been solved under separate projects using the integral representation for only the kinematic aspect of the problem. These problems include viscous flows past stationary and oscillating airfoils. Under the present project, the kinetic aspect of the unsteady flow problem has been recast into an integral representation. New numerical procedures have been developed for this new formulation of the entire unsteady problem as a set of integral representations. Numerical results have been obtained for test problems involving time-dependent flows past finite flat plates and circular cylinders. The computational advantages of this approach have been conclusively demonstrated.

In addition to the five specific tasks originally proposed, efforts have been devoted to the generalization of the integral-representation approach in studies of turbulent and compressible flows. The overall activities carried out at Georgia Tech in developing the integral-representation approach is represented by the list of articles given in this report.

MOST SIGNIFICANT RESEARCH ACCOMPLISHMENT

Under this project, a new approach based on the integral-representations of flow variables has been fully developed for the numerical solution of steady incompressible viscous flows. Several unique attributes of this approach has been demonstrated conclusively and implemented successfully. The approach has been utilized to solve various problems of practical importance. It has received recognization within the fluid dynamics community as one of the most advanced approaches for viscous flow problem.

In spite of recent progress in computational methods, many flow problems involving appreciable regions of flow separation are beyond the scope of conventional finite-difference and finite-element methods. The limitation of these conventional methods is especially acute for high Reynolds number separated flows. This limitation is a direct consequence of at least three major obstacles: (1) the excessive computer time and data storage needs for the solution of the Navier-Stokes and continuity equations, (2) the difficulties and uncertainties associated with the numerical treatment of certain boundary conditions, and (3) the lack of accurate and general methods of describing turbulent transport phenomena which occur in separated flows at large Reynolds numbers.

The integral-representation approach transcends the limitation of the conventional finite-difference and finite-element methods because of a distinguishing feature it possesses. It allows the velocity (or stream function) values to be computed explicitly, point by point, rather than implicitly. This feature offers several highly advantageous attributes: (a) the solution field can be confined to the viscous region of the flow, (b) the confined solution can be segmented and each segment treated independently of other segments, and (c) numerical boundary conditions that presented difficulties in conventional methods can be treated in a precise manner. Detailed discussions of the distinguishing feature and the advantageous attributes it offers are presented in published articles (See attached list).

In the earlier stage of development of the integral-representation approach (Articles 1 to 5), the emphasis of research has been focused on the demonstration of the validity of the new concepts involved. Detailed investigations of the various advantages offered by this approach constitute this stage of development. In particular, it has been shown that the approach is useful under quite general circumstances, for compressible as well as steady flows, two-dimensional as well as three-dimensional flows, potential as well as viscous flows (Article 9).

The superior solution efficiency and accuracy has been firmly established (Articles 13, 17). It was shown by numerical illustrations that a factor of twenty, or more, reduction in computing needs, from that associated with previous methods, are obtained for problems involving complex boundary geometries. The first obstacle, that of excessive computing needs, is removed for two-dimensional flow problems by the integral-representation approach. In fact, a standardized, user oriented, code of computer program has been made available for steady internal flows (Articles 13, 20). The input required for this program is extremely simple. The computer time required for a problem of reasonable complexity in two-dimensions is only a few minutes on the CDC-6600 computer.

The second obstacle mentioned above, that associated with the numerical treatment of boundary conditions, is also removed by the integral-representation approach. Detailed discussions about this problem are given in Article 11.

The third obstacle, that of lack of suitable methods of describing turbulent transport phenomena, partly results from the first two obstacles. That is, the lack of highly efficient numerical procedures has severely limited the ability of researchers to test and calibrate various proposed models of turbulence. Because of the necessarily empirical foundation of turbulence modeling, such tests and calibrations must be extensive. The superior solution speed and accuracy offered by the integral-representation approach offers an exciting opportunity in this regard. Efforts in computing turbulent flows constitute the main emphasis of future research by the principal investigator and his associates at Georgia Tech. Although these efforts were initiated only recently, very encouraging results have already been obtained for various types of turbulent flows (Articles 15, 16, 21).

PERSONNEL SUPPORTED

James C. Wu, Professor

As the principal investigator of this project, Dr. Wu directed the development of the integral-representation approach. He contributed to the theoretical aspect of this study, resolved major difficulties associated with the numerical treatment of boundary conditions, analyzed the flowfield segmentation technique, and extended the integral-representation approach to turbulent and to compressible flows.

Magdy M. Wahbah, Post-doctoral Fellow

Dr. Wahbah developed certain analytical expressions for geometric functions which are needed in the integral-representation approach. The availability of these analytical expressions made it possible to carry out the necessary computations with superior efficiency and high accuracy. He utilized these expressions to establish a standardized solution procedure for internal flows.

K. Balasubramnian, M. El Refaee, R. Kumar, and Y. M. Rizk, Graduate Research Assistants. The individual just named have been supported at various time during this project and contributed to the programming aspects of this research.

THE INTEGRAL-REPRESENTATION APPROACH

A list of pertinent articles authored by researchers at the Georgia Institute of Technology, with asterisks indicating those in which NSF support is acknowledged.

1. J. C. Wu and J. F. Thompson, "Numerical Solution of Unsteady, Three-Dimensional Navier-Stokes Equations," Proceedings Project SQUID Workshop on Fluid Dynamics of Unsteady, Three-Dimensional, and Separated Flows, pp. 253-280, Purdue University, Lafayette, Indiana, October, 1971.

2. J. C. Wu and J. F. Thompson, "Numerical Solutions of Time-Dependent Incompressible Navier-Stokes Equations Using an Integro-Differential Formulation," Vol. 1, No. 2, pp. 197-215, Journal of Computers and Fluids, 1973.

3. J. F. Thompson, S. P. Shanks, and J. C. Wu, "Numerical Solution of the Three-Dimensional Navier-Stokes Equations in Integro-Differential Form: Flow about a Finite Body," Proceedings AIAA Computational Fluid Dynamics Conference, pp. 123-132, July 1973.

4. J. F. Thompson, S. P. Shanks, and J. C. Wu, "Numerical Solution of Three-Dimensional Navier-Stokes Equations Showing Trailing Tip Vortices," AIAA Journal, Vol. 12, No. 6, pp. 787-794, June 1974.

5. J. C. Wu, "Integral Representations of Field Variables for the Finite Element Solution of Viscous Flow Problems," Proceedings of the 1974 Conference on Finite Element Methods in Engineering, pp. 827-840, Clarendon Press, 1974.

6. J. C. Wu, "Division of Computation Field for the Finite Element Method", Proceedings of International Symposium on Finite Element Method in Flow Problems, pp. 767-770, University of Alabama at Huntsville Press, 1974.

7. J. C. Wu, A. H. Spring, and N. L. Sankar, "A Flowfield Segmentation Method for the Numerical Solution of Viscous Flow Problems," Proceedings of the Fourth International Conference on Numerical Methods in Fluid Dynamics, Lecture Notes in Physics, Vol. 35, pp. 452-457, Springer-Verlag, 1975.

8. J. C. Wu, "Velocity and Extraneous Boundary Conditions of Viscous Flow Problems," AIAA Paper No. 75-47, American Institute of Aeronautics and Astronautics, 1975.

*9. J. C. Wu, "Finite Element Solution of Flow Problems Using Integral Representation," Proceedings of Second International Symposium on Finite Element Methods in Flow Problems, International Centre for Computer Aided Design, Conference Series No. 2/76, pp. 205-216, June, 1976.

10. J. C. Wu and S. Sampath, "A Numerical Study of Viscous Flows Around Airfoils," AIAA paper 76-337, American Institute of Aeronautics and Astronautics, 1976.

*11. J. C. Wu, "Numerican Boundary Conditions for Viscous Flow Problems," AIAA Journal, Vol. 14, No. 8, pp. 1042-1049, 1976.

12. J. C. Wu and N. L. Sankar, "Explicit Finite Element Solution of the Viscous Flow Problem," Proceedings of the 1976 International Conference on Finite Element Methods in Engineering, University of Adelaide, Australia, 1976.

*13. J. C. Wu and M. Wahbah, "Numerical Solution of Viscous Flow Equations Using Integral Representations," Proceedings of the Fifth International Conference on Numerical Methods in Fluid Dynamics, Lecture Series in Physics, Springer-Verlag, Vol. 59, pp. 448-453, 1976.

*14. J. C. Wu, "Prospects for the Numerical Solution of General Viscous Flow Problems," Proceedings of the Lockheed-Georgia Company Viscous Flow Symposium, LGTTER0044, pp. 39-104, 1976.

15. J. C. Wu and A. Sugavanam, "A Method for the Numerical Solution of Turbulent Flow Problems," AIAA Paper No. 77-649, Proceedings of AIAA 3rd Computational Fluid Dynamics Conference, pp. 168-177, 1977; also to appear in AIAA Journal, Vol. 16, No. 9, Sept. 1978.

16. J. C. Wu, M. Wahbah, and A. Sugavanam, "Some Numerical Solutions of Turbulent Flow Problems by the Use of Integral Representations," Proceedings of Symposium on Applications of Computer Methods in Engineering, Vol. 11, pp. 983-992, University of Southern California, 1977.

17. J. C. Wu, N. L. Sankar, and S. Sampath, "A Numerical Study of Unsteady Viscous Flows Around Airfoils," Proceedings of AGARD Symposium on Unsteady Aerodynamics, AGARD-CP-227, pp. 24-1-18, February 1978.

18. J. C. Wu, "Prospects for Computational Aerodynamics," Proceedings of NASA Workshop on Future Computer Requirements for Computational Aerodynamics, NASA CP-2032, February, 1978.

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RELATED EVENTS

During the past three years, the principal investigator has been invited by seven different governmental, academic, and industrial organizations to give invited lectures, seminars, and to serve as panelists at specialists meetings. These invitations resulted from the impact that the integral-representation approach, which is the focal point of this research project, has made to the field of computational fluid dynamics. The approach is now recognized by many fluid dynamicists as a most promising one for the future.

In addition to the above, the principal investigator has been asked to serve as a reporter and a reviewer for international technical conferences, a reviewer of technical articles for several journals, a reviewer of proposals for several governmental agencies, a U.S. delegate to two International Symposiums supported by branches of the U.S. government, and a collaborator in a NATO research project. The approach has been utilized by a number of researchers elsewhere, in academic and research organizations, to solve various viscous flow problems of interest.